4-8 Study Guide and Notes

**Triangles and Coordinate Proof**

**Position and Label Triangles** A coordinate proof uses points, distances, and slopes to prove geometric properties. The first step in writing a coordinate proof is to place a figure on the coordinate plane and label the vertices. Use the following guidelines.

1. Use the origin as a vertex or center of the figure.
2. Place at least one side of the polygon on an axis.
3. Keep the figure in the first quadrant if possible.
4. Use coordinates that make computations as simple as possible.

**Example:** Position an isosceles triangle on the coordinate plane so that its sides are \(a\) units long and one side is on the positive \(x\)-axis.

Start with \(R(0, 0)\). If \(RT\) is \(a\), then another vertex is \(T(a, 0)\). For vertex \(S\), the \(x\)-coordinate is \(\frac{a}{2}\). Use \(b\) for the \(y\)-coordinate, so the vertex is \(S\left(\frac{a}{2}, b\right)\).

**Exercises**
Name the missing coordinates of each triangle.

1. \(P(2, 2)\) \(R(0, 0)\) \(B(2p, 0)\)
2. \(T(?) (2a, ?a)\)
3. \(F(?) (?, ?)\)

**Position and label each triangle on the coordinate plane.**

4. Isosceles triangle \(\triangle RST\) with base \(RS\) 4\(a\) units long
5. Isosceles right \(\triangle DEF\) with legs \(e\) units long
6. Equilateral triangle \(\triangle EQI\) with vertex \(Q(0, \sqrt{3}b)\) and sides 2\(b\) units long
4-8 Study Guide and Intervention (continued)

**Triangles and Coordinate Proof**

**Write Coordinate Proofs** Coordinate proofs can be used to prove theorems and to verify properties. Many coordinate proofs use the Distance Formula, Slope Formula, or Midpoint Theorem.

**Example:** Prove that a segment from the vertex angle of an isosceles triangle to the midpoint of the base is perpendicular to the base.

First, position and label an isosceles triangle on the coordinate plane. One way is to use $T(a, 0), R(-a, 0),$ and $S(0, c)$. Then $U(0, 0)$ is the midpoint of $RT$.

**Given:** Isosceles $\triangle RST$; $U$ is the midpoint of base $RT$.

**Prove:** $SU \perp RT$

**Proof:**

$U$ is the midpoint of $RT$ so the coordinates of $U$ are \( \left( \frac{-a + a}{2}, \frac{0 + 0}{2} \right) = (0, 0) \). Thus $SU$ lies on the $y$-axis, and $\triangle RST$ was placed so $RT$ lies on the $x$-axis. The axes are perpendicular, so $SU \perp RT$.

**Exercise**

**PROOF** Write a coordinate proof for the statement.

Prove that the segments joining the midpoints of the sides of a right triangle form a right triangle.

The midpoint $E$ of $BC$ is \( \left( \frac{0 + 2m}{2}, \frac{2m + 0}{2} \right) = (m, m) \)

The midpoint $F$ of $AC$ is \( \left( \frac{0 + 2m}{2}, \frac{0 + 0}{2} \right) = (m, 0) \)

The midpoint $D$ of $AB$ is \( \left( \frac{0 + 0}{2}, \frac{0 + 2m}{2} \right) = (0, m) \)

The slope of $DE = \frac{m - m}{m - 0} = 0$.

The slope of $EF = \frac{m - 0}{m - m} = \text{undefined}$.

The slope of $DF = \frac{m - 0}{m - m} = \text{undefined}$.

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